# Time value of money and bond pricing

* Net present value = Present value − Cost

  If NPV > 0 ⇒ buy, otherwise don't buy

* Future value of an investment, $FV = C_0 (1+r)^T$

* Present value, $PV = \dfrac{C_T}{(1+r)^T}$.

* Present value of a cash flow, $PV = \dfrac{C_1}{1+r} + \dfrac{C_2}{(1+r)^2} + \dots$

$$NPV = -C_0 + \frac{C_1}{1+r} + \frac{C_2}{(1+r)^2} + \dots \frac{C_T}{(1+r)^T} = -C_0 + \sum_{1}^{T} \frac{C_i}{(1+r)^i}$$

* Annual rate of return = Effective annual interest rate = Effective ~~an~~ annual yield $= \left(1+\frac{r}{m}\right)^m - 1$

  where r is stated annual interest rate.

  (-) Effective rate > stated rate due to compounding

* Perpetuity: eg. British bonds called consols

  For a consol that pays coupon C each year

$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \frac{C}{(1+r)^3} + \dots$$

$$\boxed{PV = \frac{C}{r}}$$

* Growing perpetuity: Coupon rising at g% every year

$$PV = \frac{C}{1+r} + \frac{C(1+g)}{(1+r)^2} + \frac{C(1+g)^2}{(1+r)^3} + \dots + \frac{C(1+g)^{N-1}}{(1+r)^N} + \dots$$

$$\boxed{PV = \frac{C}{r-g}}$$

## ☆ Annuity:

| | Now 0 | 1 | 2 | 3 | T | T + 1 | T + 2 | |
|---|---|---|---|---|---|---|---|---|
| End of year | 0 | 1 | 2 | 3 | T | T + 1 | T + 2 | |
| Consale 1 | | C | C | C ... | C | C | C | ... |
| Consale 2 | | | | | | C | C | ... |
| Annuity | | C | C | C ... | C | | | |

$$PV_{c1} = \frac{C}{r} \quad , \quad V_{T, c2} = \frac{C}{r} \quad \Rightarrow \quad PV_{c2} = \frac{C}{r(1+r)^T}$$

$$\therefore \; PV_{annuity} = \frac{C}{r}\left[1 - \frac{1}{(1+r)^T}\right]$$

$$= \underbrace{C}_{\text{Periodic payment}} \underbrace{\left[\frac{1}{r} - \frac{1}{r(1+r)^T}\right]}_{\text{Annuity factor}}$$

☆ Growing annuity:

$$PV = C\left[\frac{1}{r-g} - \frac{1}{r-g}\left(\frac{1+g}{1+r}\right)^T\right]$$

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☆ Pure discount bonds - single payment at a fixed future date; payment at maturity is termed bond's face value

(-) aka zero-coupon bonds / zero / bullet / discount

(-) $PV = \frac{F}{(1+r)^T}$

☆ Level-coupon bond - coupon, C, paid every 6 months + face value, F paid at maturity

↳ aka principal / denomination

$$PV = \frac{C}{1+r} + \frac{C}{(1+r)^2} + \dots \frac{C}{(1+r)^T} + \frac{F}{(1+r)^T}$$

☆ Consol → an example is preferred stock

☆ Yield to maturity → discount rate that equates the price of the bond with the discounted value of the coupons and face value

## ✩ Present value of common stocks

(·) Value of a firm's common stock to the investor is equal to the present value of all of the expected future dividends

$$P_0 = \frac{Div_1}{1+r} + \frac{Div_2}{(1+r)^2} + \dots = \sum_{1}^{\infty} \frac{Div_t}{(1+r)^t}$$

(·) Case 1 : Zero growth ⇒ constant dividend

$$P_0 = \frac{Div}{r}$$

Case 2 : Constant growth ⇒ dividend grows at rate $g$

$$P_0 = \frac{Div}{r-g}$$

Case 3 : Differential growth

## ✩ Estimates of parameters in the dividend - discount model

(·) How to calculate $g$

Earnings next year = Earnings this year + Retained earnings this year × Return on retained earnings

$$\Rightarrow \frac{\text{Earnings next year}}{\text{Earnings this year}} = 1 + \frac{\text{Retained earnings}}{\text{Earnings}} \times \text{Return on retained earnings}$$

$\Rightarrow 1 + g = 1 + \text{Retention ratio} \times \text{Return on retained earnings}$

$\Rightarrow g = \text{Retention ratio} \times \text{Return on Equity}$

(·) How to calculate $r$

$$P_0 = \frac{Div}{r-g} \Rightarrow r = \underbrace{\frac{Div}{P_0}}_{\text{Dividend yield}} + \underbrace{g}_{\text{growth rate of dividends}}$$

✩ Company is called a cash cow if it pays all of the earnings out to stockholders as dividends

$\Rightarrow$ EPS (Earnings per share) = Div (Dividend per share)

$$\Rightarrow \frac{EPS}{r} = \frac{Div}{r}$$

✩ Stock price if firm commits to a new project

$$= \frac{EPS}{r} + NPVGO$$

↳ net present value per share of growth opportunity

## → Indian g-secs :-

* Treasury Bills → presently in 3 tenors, 91 day, 182 day & 364 day
  → zero coupon secs and pay no interest
  → issued at a discount & redeemed at face value

(·) How is yield calculated ?

$$\text{Yield} = \left(\frac{\overset{F}{100} - P}{P}\right)\left(\frac{365}{D}\right) \times 100$$

face value, F

$P \rightarrow$ purchase price, $D \Rightarrow$ days to maturity

Derivation : $P + (y\%\text{ per annum})P = F$

$$\Rightarrow P\left(1 + \frac{y}{100} \times \frac{D}{365}\right) = F$$

$$\Rightarrow \frac{y}{100} \times \frac{D}{365} = \frac{F}{P} - 1$$

$$\Rightarrow \frac{y}{100} = \left(\frac{F-P}{P}\right)\left(\frac{365}{D}\right)$$

$$\Rightarrow y = \left(\frac{F-P}{P}\right)\left(\frac{365}{D}\right) \times 100$$

* Cash Mgt Bills → new short-term instrument introduced in 2010
  → generic character of T-Bills but issued for maturity < 91 days

* Dated g-ses → carry fixed or floating coupon which is paid on face value on half yearly basis
  → Tenor ranges from 5 to 40 years
  → Public Debt Office (PDO) of RBI acts as a registry / depository of g-secs
  → Nomenclature of typical dated fixed coupon g-sec 7.49 % GS 2017

Coupon : 7.49 % on face value / year
Maturity : 2017 Apr 16
Issue : 2007 Apr 16
Coupon payment dates : Half yearly (Oct 16 & Apr 16)
Min. amt. of issue : Rs. 10,000

(5)

→ Each security is assigned a unique number called: IS IN (International Security ID Number)

→ If coupon payment date is on Sun or holiday → coupon payment made on next working day

If ~~coupon~~ maturity date falls on Sun or holiday → redemption proceeds paid on pvs working day.

☆ Coupon yield = $\frac{\text{Coupon payment}}{\text{Face value}}$

Current yield = $\frac{\text{Annual coupon rate}}{\text{Purchase price}} \times 100$

Yield to maturity (YTM) → expected rate of return on a bond if it is held until maturity

→ discount rate which equates the present value of future cash flows from a bond to its current market price

→ internal rate of return on the bond

☆ Day count convention in India for arriving at holding period

Bond Mkt → Days in month = 30, days in year = 360

Money Mkt → Days in month = actual, days in year = 365

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☆ Macaulay duration of a bond is a measure of time taken to recover the initial investment in present value terms

(•) Calculation

① Each future cash flow is discounted to its resp PV for each period (1 period = 6 months ∵ coupon paid out every 6 months)

② PVs are multiplied with their resp. time periods (PV of 1st coupon × 1, 2nd coupon × 2 ...)

③ Above weighted PVs are added and divided by current price (i.e. sum of PVs found in step 1)

④ Result will give no. of periods. This is divided by 2 to get no. of years.

(-) Duration always ≤ life to maturity of bond

(-) Only for zero coupon bond is duration = maturity

(-) Duration is useful primarily as a measure of the sensitivity of a bond's market price to interest rate movements

e.g. a 15 year bond with duration of 7 years would fall approx. 7% in value if interest rate ↑ by 1% per annum.

☆ Modified duration → change in value of security to 1% change in interest rates (yield)

$$MD = \frac{\text{Macaulay duration}}{1 + \frac{YTM}{\text{no. of coupon periods in a year}}}$$

☆ PV 01 → actual change in price of a bond if yield changes by 1 basis point (0.01%)

eg. if MD = 1.78 ⇒ for 1% change in yield, value of security will change by 1.78% $= \frac{1.78}{100} \times 102 = 1.81$ Rs.

where current price = Rs. 102

∴ PV 01 $= \frac{1.81}{100}$ Rs. = 1.81 paise

∴ If yield of bond with MD = 1.78 years moves from 9 to 9.05% (5 basis pts), price of bond moves from 102 Rs. to Rs. $\left(102 - \frac{5 \times 1.81}{100}\right)$ = Rs. 101.91

☆ Convexity → Change in duration of bond per unit change in yield of bond.

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☆ Macaulay's duration $= \sum_{t=1}^{n} \frac{(PV)(CF_t) \times t}{\text{mkt price of bond}}$

## → Balance Sheet :-

### ✩ Account form

| Liabilities | Assets |
|---|---|
| Share capital | Fixed assets |
| Reserves & surplus | Investments |
| Secured loans | Current assets, loans & advances |
| Unsecured loans | Misc. expenditures & losses |
| Current liabilities & provisions | |

### ✩ Report form

I → Source of funds

1. Shareholders' funds
   a) Share capital
   b) Reserves & surplus
2. Loan funds
   a) Secured loans
   b) Unsecured loans

II → Application of funds

1. Fixed assets
2. Investments
3. Current assets, loans & advances
   Less : current liabilities & provisions
   Net current assets
4. Misc expenditure & losses

## ☆ EMI calculation :-

(-) Principal borrowed $= L$

Interest $= i\%$ per annum

Period $= n$ months

EMI $= E$

(.) $$E = \frac{L\left(\frac{i}{1200}\right)\left(1+\frac{i}{1200}\right)^n}{\left(1+\frac{i}{1200}\right)^n - 1}$$

(-) Derivation :

↳ Sum owed at the end of 1st month $= L + L\left(\frac{i}{1200}\right)$

$$= L\left(1+\frac{i}{1200}\right) = Lr$$

where $r = 1 + \frac{i}{1200}$

↳ Sum paid at the end of 1st month $= E$

$\Rightarrow$ Amount owed $= L_1 = Lr - E$

↳ Sum owed at the end of 2nd month $= L_1\left(1+\frac{i}{1200}\right) = L_1 r$

After paying $E \Rightarrow L_2 = L_1 r - E = (Lr - E)r - E = Lr^2 - E(1+r)$

↳ $||^{ly}$ at the end of $n^{th}$ month, after paying $E$, sum owed is

$$L_n = Lr^n - E(1 + r + r^2 + \dots + r^{n-1})$$

$$L_n = Lr^n - E\left(\frac{r^n - 1}{r - 1}\right)$$

↳ $\because$ payment ends after $n$ months

$$\Rightarrow L_n = 0 \Rightarrow E = \frac{Lr^n(r-1)}{r^n - 1}$$

$$\Rightarrow E = \frac{L\left(\frac{i}{1200}\right)\left(1+\frac{i}{1200}\right)^n}{\left(1+\frac{i}{1200}\right)^n - 1}$$

(.) Out of $K^{th}$ month EMI, $r^{K-1}\left[E - L(r-1)\right]$ is deducted towards principal payment and rest towards interest

# → Cost of capital :-



## ☆ Debt

(-) Cost of debt = Interest paid − tax (∵ interest is tax-deductible expenditure)

(-) Redeemable debt → repaid after a specific period

(-) Irredeemable debt → perpetual / not repaid; only interest is paid regularly

## ☆ Cost of irredeemable debt

↳ Interest = Before tax cost of debt

↳ $K_d = \frac{I}{NP}(1 - t)$

↳ in %

I : annual interest payment

NP : Net proceeds from issue of bond

## ☆ Cost of redeemable debt

(-) Take avg of Sale value (SV) & Redeemable Value (RV) while calculating cost.

(-) Before tax cost, $K_d = \dfrac{I + \frac{RV - SV}{n}}{\frac{RV + SV}{2}}$

n → term of debt till maturity

(-) After tax cost, $K_d = \left(\dfrac{I + \frac{RV - SV}{n}}{\frac{RV + SV}{2}}\right)(1 - t)$

RV = Face value, SV = Price

## ☆ Cost of irredeemable preference share

$K_p = \dfrac{D_p}{NP}$

$D_p$ = preference dividend

NP = net proceeds from issue of preference shares

## ☆ Cost of redeemable preference shares

$$K_p = \dfrac{D_p + \frac{1}{n}(RV - NP)}{\frac{RV + NP}{2}}$$

## ☆ Cost of equity share capital

(.) Dividend Price Approach → cost of equity is the rate of return that the shares are expected to earn in the form of future dividends

$$K_e = \frac{D}{MP}$$

D → dividend per share
MP → Market price per share

(.) Investopedia, $K_e = \frac{D}{MP}$ + growth rate of dividends

(.) CAPM, Cost = Risk free rate of return + β(Market rate of return − Risk free rate of return)

## ☆ Cost of ~~re~~ retained earnings

$$K_r = K_e(1-t)(1-c)$$

where $K_e$ = cost of equity / reqd rate of return of shareholders

c = Brokerage, commission etc.

## ☆ Weighted Average Cost of Capital (WACC)

$$= \frac{MV_e \times R_e + MV_d \times R_d(1-t)}{MV_e + MV_d}$$

MV = Market value
d → debt
e → equity

→ Other formulae :-

☆ Approximation for YTM, YTC

(-) $YTM, i, IRR = \dfrac{Coupon + \dfrac{Face\ Value - Price}{n}}{\dfrac{Face\ value + Price}{2}}$

(-) $YTC\ (Yield\ to\ call) = \dfrac{Coupon + \dfrac{Call\ price - Mkt\ price}{n}}{\dfrac{Call\ price + Mkt\ price}{2}}$

☆ $Holding\ period\ return\ (HPR) = \dfrac{Interest\ income + (End\ of\ period\ value - Initial\ value)}{Initial\ value}$

(-) $Annualised\ HPR = (HPR)^{1/years} - 1$

☆ $Number\ of\ bonds\ bought = \dfrac{Cash}{Nominal\ value \times Dirty\ price}$

Dirty price is the price of a bond including the interest accrued

☆ $Times\ interest\ earned\ ratio = \dfrac{EBIT}{Interest\ expenses}$

☆ $Capital\ gearing\ ratio = \dfrac{Common\ stockholders\ equity}{Fixed\ cost\ bearing\ funds}$

Fixed cost bearing funds = Debentures + Preference share capital + Other long term loans

☆ Margin of safety = Current output − Break even output

$MoS\ \% = \dfrac{MoS}{Current\ output} \times 100\%$

☆ $Debt\ Service\ Coverage\ Ratio = \dfrac{Operating\ income}{Total\ Debt\ service\ costs}$

$DSCR = \dfrac{PAT + Annual\ interest + lease\ rental + non\text{-}cash\ expenses}{Installment\ (annual\ interest + principal\ repayment) + Lease\ rental}$

↳ Ratio of cash available for debt servicing to interest,
(12) principal and lease payments

↳ High ratio is good

↳ ~~Cash~~ ~~prof~~ $DSCR = \dfrac{\text{Net operating income}}{\text{Debt service}}$

Net operating income (Net income + Depreciation + Interest expense + other non-cash items)

Debt service = Principal repayment + Interest payment + Lease payments

☆ Dupont analysis to assess ROE (Return on Equity)

(.) 3 major financial metrics drive ~~Return~~ ROE = Operating efficiency, asset use efficiency and financial leverage

$\text{Operating } \eta = \dfrac{\text{Net income}}{\text{Average shareholders' equity}}$

$\text{Asset use } \eta = \text{Total asset turnover} = \text{Asset turnover ratio}$

$= \dfrac{\text{Sales}}{\text{Avg total assets}}$

(.) ROE = Return on assets × leverage

(.) $\dfrac{\text{Net income}}{\text{Total equity}} = \dfrac{\text{Net income}}{\text{Sales}} \times \dfrac{\text{Sales}}{\text{Avg total assets}} \times \dfrac{\text{Avg total assets}}{\text{Equity}}$

☆ Walter's model, dividend

$$\text{Market value} = \dfrac{\text{Dividend} + \dfrac{\text{ROI (Earnings} - \text{Dividend)}}{\text{Capitalization rate}}}{\text{Capitalization rate}}$$

☆ Solvency ratio $= \dfrac{\text{Outside liabilities}}{\text{Total assets}}$

Outside liabilities = Debenture + Overdraft + Creditors

★ Black-scholes model for option pricing :

(-) Assumptions : European option (can only be exercised at expiration), no dividends paid out, efficient mkt (movement cannot be predicted), no txn costs in buying option, risk free rate & volatility of underlying are known & constant

(.) Uses : current stock prices, expected dividends, option's strike price, expected interest rates, time to expiration & expected volatility

(-) To find : theoretical value of European style options

★ Modigliani Miller approach,

$$\text{Value of firm} = \frac{EBIT}{K_o}(1-t)$$

where $t \rightarrow$ tax rate ; $K_o$ = overall cost of capital aka capitalization rate.

★ Capital gearing ratio = $\dfrac{\text{Fixed income bearing secs}}{\text{Equity share capital + Reserves}}$

Fixed income bearing secs = Preference share capital + Debentures

★ Currency deposit ratio = money held by public in currency to that they hold in bank deposits.